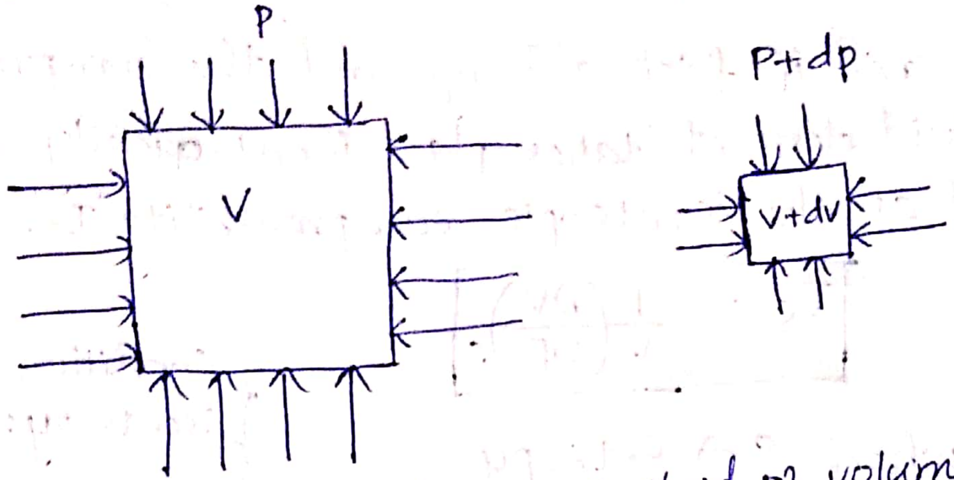
11/8/20

MODULE-I

* Compressibility:

- The amount by which a substance can be compressed is given by a specific property of the substance called Compressibility.
- All real substances are compressible, when squeezed or pressed on them, their density will change.



- Consider a small element of fluid of volume 'V'. The pressure exerted on the sides of the element is 'p'. Assume the pressure is increased by an infinitesimal amount dp. The volume of the element will change by the amount dv. The volume is decreased. So dv is '-ve' quantity.
- By the definition, the compressibility 'τ' of the fluid:

$$\boxed{\tau = -\frac{1}{V}\frac{dv}{dP}} \longrightarrow ①$$

- Physically, the compressibility is the fractional change in volume of the fluid element per unit change in pressure.
- When a gas is compressed, its temperature tends to ↑se, depending upon the amount of heat transferred

into or out of the gas through the boundaries of the system.

- If the temperature of the fluid element is held constant, then 'τ' is defined as the isothermal compressibility 'τ_T'.

$$\boxed{\tau_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T}$$

- If no heat is added to or taken away from the fluid element and if friction is ignored, the compression of the fluid element takes place isentropically and 'τ' is identified as the isentropic compressibility 'τ_S'.

$$\boxed{\tau_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S}$$

where, $S \Rightarrow$ Entropy.

{ Conditions for isentropy: (i) Adiabatic. (ii) Reversible.

- The value of 'τ_T' and 'τ_S' are different for gases and liquids. The compressibility of gases are several orders of magnitude larger than that of liquids.
- Define v as the specific volume (volume/unit mass).

Hence $v = \frac{1}{\rho}$. The 'τ' is:

$$\left[\begin{array}{l} \rho = \frac{m}{V} \\ V = \frac{m}{\rho} \\ v = \frac{1}{\rho} \end{array}\right]$$

$$\tau = \frac{1}{\rho}\frac{d\rho}{dP}$$

$$\therefore \boxed{d\rho = \rho\tau\, dP}$$

Consider the flow over an airfoil.

Case 1:

(i) If the fluid is liquid:

Compressibility 'τ' is very small, so $d\rho$ will be negligibly small. So assuming that 'ρ' is constant and the flow of the liquid is incompressible.

(ii) If the fluid is gas:

Compressibility 'τ' is large. So '$d\rho$' can be large. Thus, 'ρ' is not constant and in general, the flow of a gas is a compressible flow.

(iii) For low-speed flow of gas:

'dp' is small even though 'τ' is large, the value of $d\rho$ can be dominated by the small dp. In such cases ρ is constant and analyze low-speed gas flows as incompressible flows.

⇒ The most convenient index to gage whether a gas flow can be considered incompressible or whether it must be treated as compressible is the mach number 'M'.

$$\boxed{M = \frac{V}{a}}$$; where, $V \rightarrow$ local flow velocity.
$a \rightarrow$ local speed of sound.

when $M > 0.3$, the flow is compressible.

*24/8/20 Mach Cone and Mach Angle:

Mach Cone:

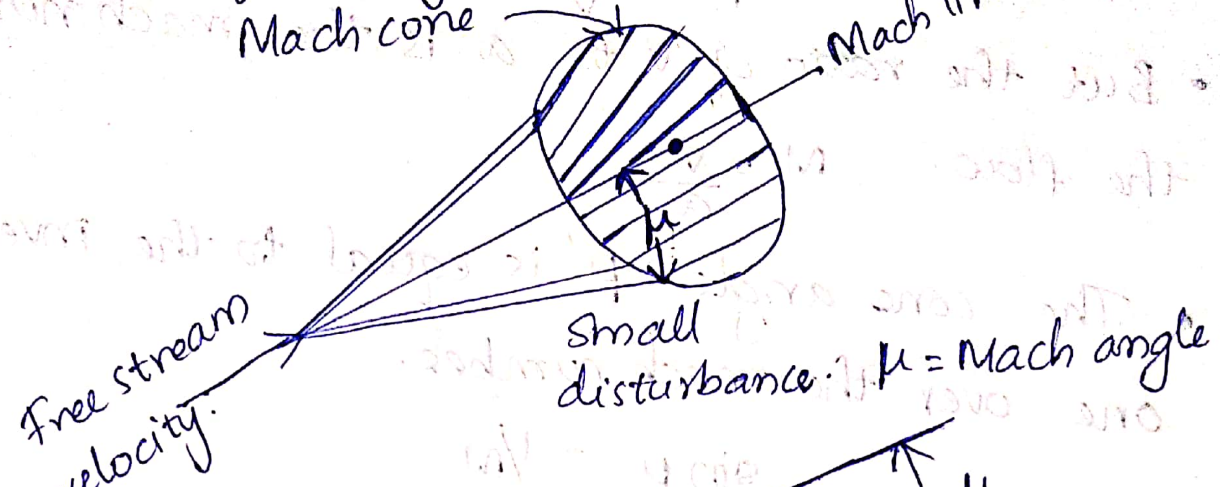
→Sonic boom, shock wave that is produced by an aircraft or other object flying at a speed equal to or exceeding the speed of sound and that is heard on the ground as a sound like a clap of thunder.

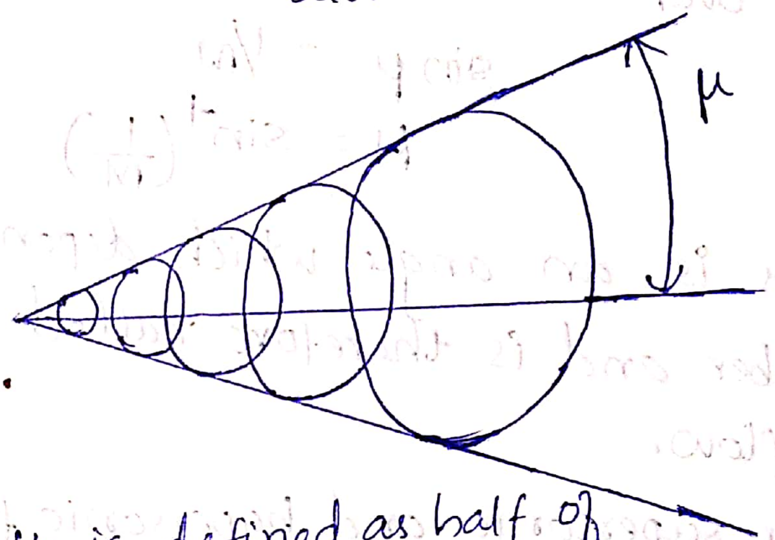
When an aircraft travels at subsonic speed, the pressure disturbances, or sounds, that it generates extend in all directions. Because this disturbance is transmitted earthward continuously to every point along the path, there are no sharp disturbances or changes of pressure. At supersonic speeds, however, the pressure field is confined to a region extending mostly to the rear and extending from the craft in a restricted widening cone called a Mach cone. As the aircraft proceeds, the trailing parabolic edge of that cone of disturbance intercepts the Earth, producing on Earth a sound of a sharp bang or boom. When such an aircraft flies at low altitude, the shockwave may be of sufficient intensity to cause glass breakage and other damage. The intensity of the sonic boom is determined not only by the distance between the craft and the ground but also by the size and shape of the aircraft, the types of maneuvers that it makes, and the atmospheric pressure, temperature, and winds. If the aircraft is especially long double sonic booms might be detected, one emanating from the leading edge of the plane and one from the trailing edge.

* The series of wave fronts form a disturbance envelope given by a straight line which is tangent to the family of circles. * It will be seen that all the disturbance waves lie within a cone having a vertex or apex at the body at time considered.

* The locus of all the leading surfaces of the waves of this cone is known as <u>Mach cone</u>.

* Whenever there is any disturbance in a compressible fluid, the disturbance is propogated in all the directions with a velocity of sound.
* The nature of propogation depends on the Mach number.





Mach Angle: It is defined as half of the angle of Mach cone. It is denoted by 'μ'.

$$\sin \mu = \frac{1}{M}$$

⇒ **Zone of action**: • All disturbances confine inside the Mach cone extending downstream of the moving body is called as zone of action.

⇒ **Zone of Silence**: • The region outside the Mach cone and extending upstream is known as zone of silence.

• The pressure disturbances are largely concentrated in the neighbourhood of the Mach cone that forms the outer limit of the zone of action.

- From trigonometry the sine of the cone angle 'μ' is equal to the ratio of 'a' and 'v.'

$$\sin \mu = \frac{a}{V}$$

- But the ratio of V to a is the mach number of the flow. $M = \frac{V}{a}$

- The cone angle 'μ' is equal to the inverse sine of one over the mach number.

$$\sin \mu = 1/M$$
$$\mu = \sin^{-1}\left(\frac{1}{M}\right)$$

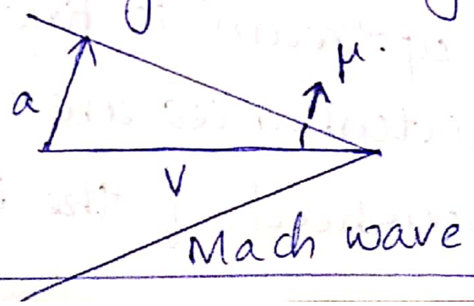
- μ is an angle which depends only on the Mach number and is therefore called the Mach angle of the flow.

- For supersonic and hypersonic flows, small disturbances are transmitted downstream within a cone.

- The sound waves strike the edge of the cone at a right angle and the speed of the sound wave is denoted by the letter 'a'.

- The edge of the cone is depicted a dimensionally by the blue lines on the figure.

- The flow is moving at velocity 'v' which is greater than 'a'.



Problem 1:

Calculate the mach angle for an object moving with a velocity of 480 m/s. If the ambient condition is 1 bar and 288 K.

Sol: Given data:

velocity of the object, $V = 480\,m/s$

we know, $\sin\mu = \frac{1}{M}$

$\therefore$ Mach angle, $\mu = \sin^{-1}\left(\frac{a}{V}\right)$

$\therefore \mu = \sin^{-1}\left(\frac{340.197}{480}\right) = \underline{\underline{45.132^\circ}}$

$\boxed{\text{Speed of sound},\ a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287.04 \times 288} = \underline{\underline{340.197\,m/s}}}$

Problem 2:

For an aircraft travelling in air the mach angle is found to be 30°, determine the velocity of aircraft if the temperature of air is −7°C.

Sol: Given data:

Mach angle, $\mu = 30^\circ$

Temperature, $T = -7^\circ C = -7 + 273 = 266\,K$

Speed of sound, $a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287.04 \times 266} = \underline{\underline{326.946\,\frac{m}{s}}}$

$\sin\mu = \frac{1}{M} = \frac{a}{V}$

$\therefore$ velocity of aircraft, $V = \frac{a}{\sin\mu} = \frac{326.946}{\sin(30^\circ)} = \underline{\underline{653.892\,\frac{m}{s}}}$

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* **Nozzle:**

Nozzle is a device which is designed to control the direction and characteristics of the fluid flow. It has varying cross-sectional area and used to direct and modify fluid flow.

Mainly it is used to control the velocity. It also has other purposes. It converts the P.E into K.E.

⇒ Three types of Nozles are there in aerodynamics:

(i) Convergent Nozzle

(ii) divergent Nozzle

(iii) Convergent-divergent Nozzle.

* **Derivation:**

⇒ **Operating characteristics of Nozzle:**

from Euler's Equation:

$$\boxed{\frac{dP}{\rho} + VdV = 0} \longrightarrow (1)$$

this equation can be written as:

$$dP = -\rho VdV \longrightarrow (2)$$

Divide the eqn (2) by ρV^2

$$\boxed{\frac{dP}{\rho V^2} = -\frac{dV}{V}} \longrightarrow (3)$$

From Continuity Equation: (in differential form)

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

$$\frac{dA}{A} = -\frac{d\rho}{\rho} - \frac{dV}{V} \longrightarrow (4)$$

Substitute eqn (3) in eqn (4):

$$\frac{dA}{A} = -\frac{d\rho}{\rho} + \frac{dP}{\rho V^2}$$

$$= \frac{dP}{\rho V^2}\left[1 - \frac{V^2}{(dP/d\rho)}\right]$$

$$\therefore \frac{dP}{d\rho} = a^2 \text{ (square of speed of sound).}$$

substituting the above relation:

$$\therefore \frac{dA}{A} = \frac{dP}{\rho V^2}\left[1 - \frac{V^2}{a^2}\right]$$

$$\boxed{\frac{dA}{A} = \frac{dP}{\rho V^2}\left[1 - M^2\right]} \longrightarrow (5).$$

Cases:

⟹ when $M<1$ (the flow is subsonic):
There will be +ve pressure change for the +ve area change

⟹ when $M>1$ (the flow is supersonic):
There will be '-ve' pressure change for the +ve area change.

Substitute Eqn (3) in Eqn (5)

$$* \boxed{\frac{dA}{A} = -\frac{dV}{V}\left[1 - M^2\right]}.$$

It decides the geometry of the Nozzle.

Cases:

⟹ when $M<1$ (Subsonic flow)
'+ve' area changes causes '-ve' velocity change ($A\downarrow V\uparrow$)

⟹ when $M>1$ (Supersonic flow):
'+ve' change in area causes '+ve' change in velocity ($A\uparrow V\uparrow$).

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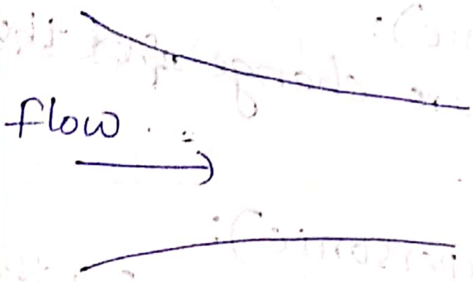
→ At subsonic speed ($M < 1$) a decrease in area increases the speed of flow.

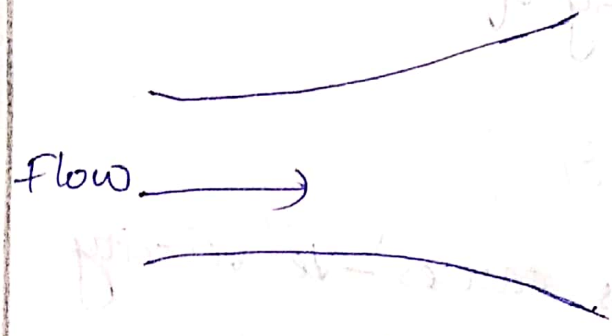
→ A subsonic Nozzle should have a convergent profile.

Nozzle [$dp < 0$, $dv > 0$]

$M < 1$ (subsonic) ($A \propto \frac{1}{V}$)



Since we need high velocity, area should be decreased. Therefore, the geometry becomes convergent for subsonic Nozzle.

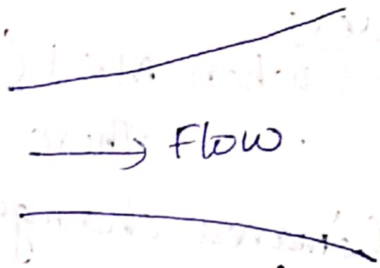


$M > 1$ (supersonic) ($A \propto V$).

Since we need high velocity, area should be increased. Therefore, the geometry becomes divergent for supersonic Nozzle.

Diffuser [$dP > 0$, $dv < 0$]

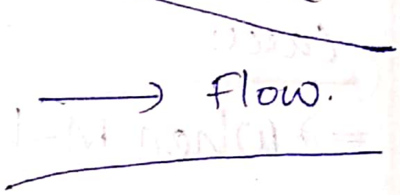
$M < 1$ ($A \propto \frac{1}{V}$)



$M < 1$ (subsonic)

Since we need to reduce the velocity in a diffuser, the area should be increased. Therefore, geometry becomes divergent for subsonic diffuser.

$M > 1$ ($A \propto V$).



$M > 1$ (supersonic).

Since we need to reduce the velocity in a diffuser, the area should also be decreased. Therefore, the geometry is convergent.

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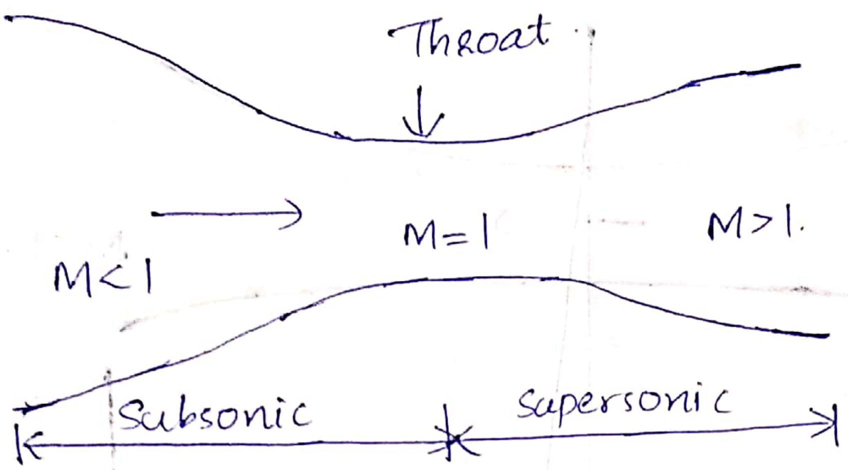
* Convergent - Divergent Nozzle: (C-D Nozzle)



when throat diameter is optimized, we obtain maximum thrust. length is not considered.

Suppose a nozzle is used to obtain supersonic flow. Stream starts with M=0 near inlet. Mach number is greater than one at exit. It is a combination of convergent and divergent section. convergent section in subsonic section and is divergent in supersonic section. It is also known as Delaval nozzle in memory of scientist Gustaf Delaval. He first used this nozzle in 19th century in steam turbines.

In both sections velocity increment is taking place.

Throat: In that throat section, area is neither increasing nor decreasing. Mach number should be unity at this section. It is the connection b/w convergent and divergent section.

$$M=1 \Rightarrow \frac{V}{a}=1 \Rightarrow V=a.$$

The velocity is called sonic velocity. It is obtained only at throat area.

$$\frac{dA}{A} = -\frac{dv}{V}(1-M^2)$$

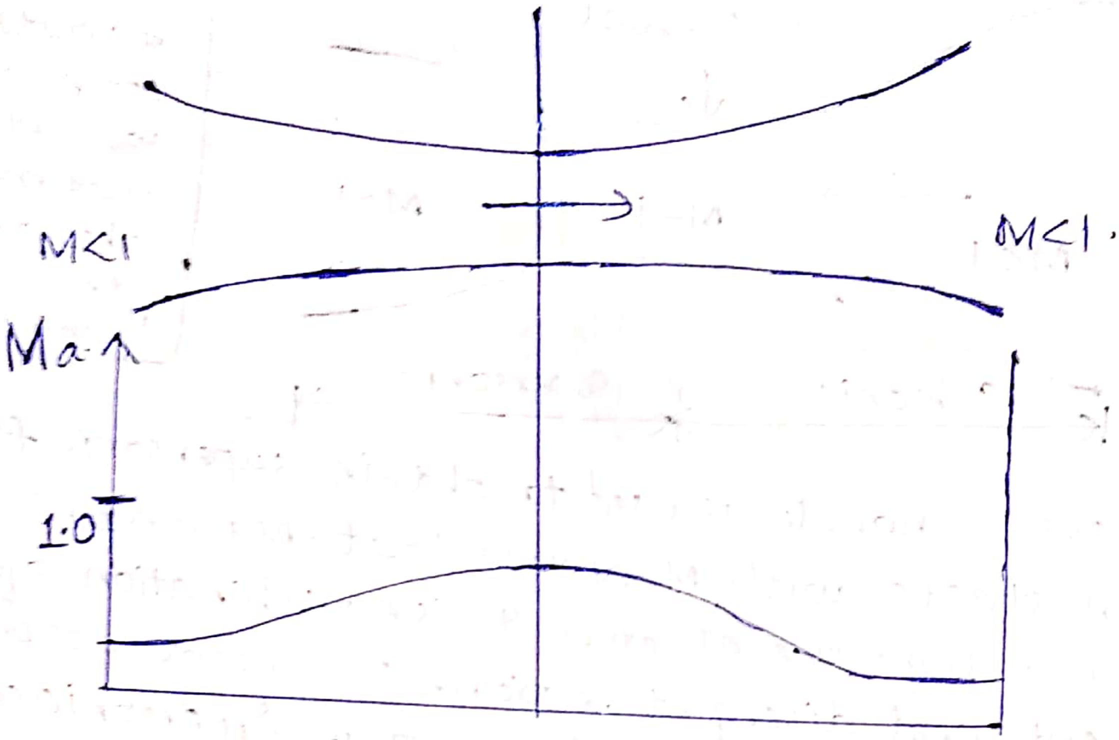
This term becomes zero

when $M=1 \Rightarrow \frac{dA}{A} = -\frac{dv}{V}\,\boxed{(1-1)}$

But this 2 terms cannot be zero.

Even though M=1 velocity will be increasing and area is constant

* Convergent - Divergent duct with $M \neq 1$ at throat



Since $M \neq 1$, it can be ither subsonic or supersonic.

when $M<1$ at inlet then it is subsonic at the exit also. (subsonic) Throughout the flow will be subsonic. Velocity will not increase. In subsonic $V \alpha \frac{1}{A}$, therefore at exit the area is divergent and velocity is less.

* Isentropic flow in a converging Nozzle:

Consider the mass flow rate of an ideal gas through a converging nozzle. If the flow is isentropic.

$$\dot{m} = \rho A V$$

$$\frac{\dot{m}}{A} = \rho V$$

$$\frac{\dot{m}}{A} = \frac{P}{RT} \times V$$

$$= \frac{P}{RT} M\sqrt{\gamma RT}$$

$P = \rho RT$

$\rho = \frac{P}{RT}$

$M = \frac{V}{a}$

$V = Ma$

$= M\sqrt{\gamma RT}$

$$= \frac{P}{\sqrt{RT}} \sqrt{r}\, m$$

$$= \frac{P}{\sqrt{T}} \sqrt{\frac{r}{R}}\, m$$

$$= \frac{P}{P_0} \times P_0 \times \sqrt{\frac{T_0}{T}} \times \sqrt{\frac{1}{T_0}} \times \sqrt{\frac{r}{R}}\, m$$ (By multiplying & dividing by P_0 and $\sqrt{T_0}$).

$$= \left(\frac{T}{T_0}\right)^{\frac{r}{r-1}} \times \sqrt{\frac{T_0}{T}} \times \frac{P_0}{\sqrt{T_0}} \times \sqrt{\frac{r}{R}}\, m$$ $\left[\because \frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{r}{r-1}}\right]$

$$= \sqrt{\frac{r}{R}} \frac{m P_0}{\sqrt{T_0}} \left(\frac{T_0}{T}\right)^{\frac{1}{2} - \frac{r}{r-1}}$$

$$= \sqrt{\frac{r}{R}} \frac{m P_0}{\sqrt{T_0}} \left(\frac{T_0}{T}\right)^{-\frac{(r+1)}{2(r-1)}}$$

$$= \sqrt{\frac{r}{R}} \frac{m P_0}{\sqrt{T_0}} \frac{1}{\left(\frac{T_0}{T}\right)^{\frac{r+1}{2(r-1)}}}$$

where $\frac{T_0}{T} = \left[1 + \frac{r-1}{2} m^2\right]$

$$\boxed{\frac{\dot{m}}{A} = \sqrt{\frac{r}{R}} \frac{m P_0}{\sqrt{T_0}} \frac{1}{\left[1 + \frac{r-1}{2} m^2\right]^{\frac{r+1}{2(r-1)}}}} \longrightarrow (A)$$ (Discharge through a convergent Nozzle).

where P_0, T_0, r and R are constants.

$\Rightarrow$ The dischagge per unit area $\frac{\dot{m}}{A}$ is a function of mach number only.

$\Rightarrow$ To get maximum mach number, differentiate eq (A) w.r.t m and equate to zero:

$$\frac{d}{dm} \frac{\dot{m}}{A} = 0 \Rightarrow \sqrt{\frac{r}{R}} \frac{P_0}{\sqrt{T_0}} \left[\frac{d}{dm}\left(M \times \left[1 + \frac{r-1}{2} M^2\right]^{-\frac{(r+1)}{2(r-1)}}\right)\right] = 0$$

$$\left[1 + \frac{r-1}{2} M^2\right]^{-\frac{(r+1)}{2(r-1)}} + M \left[-\frac{(r+1)}{2\cancel{(r-1)}} \left[1 + \frac{r-1}{2} M^2\right]^{-\frac{\cancel{2}r+1}{2(r-1)}} \times \frac{\cancel{r-1} \times 2M}{\cancel{2}}\right] = 0$$

$$\left[1 + \frac{r-1}{2} M^2\right]^{-\frac{(r+1)}{2(r-1)}} \left[1 + M\left[-\frac{(r+1)}{2} \left[1 + \frac{r-1}{2} M^2\right]^{-1} \times M\right]\right] = 0$$

$$M^2 \times \frac{-(\gamma+1)}{2}\left[\frac{1}{1+\frac{\gamma-1}{2}M^2}\right] = -1$$

$$\frac{M^2}{1+\frac{\gamma-1}{2}M^2} = \frac{-2}{-(\gamma+1)} \Rightarrow M^2(1+\gamma) = 2\left(1+\left(\frac{\gamma-1}{2}\right)M^2\right)$$

$$M^2+\cancel{\gamma M^2} = 2+\cancel{\gamma M^2}-M^2 \Rightarrow 2M^2 = 2 \Rightarrow M^2 = 1 \Rightarrow \therefore M = \pm 1$$

$$\therefore \boxed{M=1}$$

Hence discharge is maximum when $M=1$.

we know that, $V = Ma$.

By logarithmic differentiation, we get:

$$\frac{dV}{V} = \frac{dM}{M} + \frac{1}{2}\frac{dT}{T} \rightarrow ① \quad \left[\because a = \sqrt{\gamma RT}\right.$$

γ & R are constants.$\left.\right]$

we know that:

$$\frac{T}{T_0} = \left[1+\frac{\gamma-1}{2}M^2\right]^{-1}$$

By logarithmic differentiation, we get:

$$\frac{dT}{T} = -\frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)} \times \frac{\gamma-1}{\cancel{2}} \times \cancel{2}M \times \frac{dM}{M} \times M$$

(Multiplying & dividing by M]

$$\frac{dT}{T} = \frac{-(\gamma-1)M^2}{1+\frac{\gamma-1}{2}M^2}\frac{dM}{M}$$

substitute in eqn ①:

$$\frac{dV}{V} = \frac{dM}{M} + \frac{1}{2}\left[\frac{-(\gamma-1)M^2}{1+\frac{\gamma-1}{2}M^2}\right]\frac{dM}{M}$$

$$= \frac{dM}{M}\left[1 - \frac{\left(\frac{\gamma-1}{2}\right)M^2}{1+\frac{\gamma-1}{2}M^2}\right]$$

$$\frac{dV}{V} = \frac{dM}{M}\left[\frac{1}{1+\frac{\gamma-1}{2}M^2}\right] \longrightarrow ②$$

We know that: $\frac{dA}{A} = -\frac{dV}{V}\left[1-M^2\right] \longrightarrow (3)$.

Substitute Eqn (2) in Eqn (3):

$$\frac{dA}{A} = -\frac{dm}{m}\left[\frac{1}{1+\frac{\gamma-1}{2}M^2}\right]\left[1-M^2\right]$$

$$\boxed{\frac{dA}{A} = (M^2-1)\times\left[\frac{1}{1+\frac{\gamma-1}{2}M^2}\right]\frac{dm}{m}} \longrightarrow (4)$$

When machnumber, $M=1$.

$\therefore dA = 0$, $\therefore A = \text{constant}$.

$\therefore M=1$ only can be obtained at throat and this happens only when discharge is maximum. We say that the nozzle is choked. The properties at throat is called as critical properties.

Sub $M=1$ for eqn (A):

$$\frac{\dot{m}}{A^*} = \sqrt{\frac{\gamma}{R}}\frac{P_0}{\sqrt{T_0}}\frac{1}{\left(1+\frac{\gamma-1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \longrightarrow (5)$$

[A^* = critical area when $M=1$.]

Divide Eqn (5) by Eqn (A):

$$\frac{\frac{\dot{m}}{A^*} = \sqrt{\frac{\gamma}{R}}\cdot\frac{P_0}{\sqrt{T_0}}\frac{1}{\left[1+\frac{\gamma-1}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}}{\frac{\dot{m}}{A} = \sqrt{\frac{\gamma}{R}}\frac{M P_0}{\sqrt{T_0}}\frac{1}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}}$$

$$\boxed{\frac{A}{A^*} = \frac{1}{M}\left[\frac{1+\frac{\gamma-1}{2}M^2}{\left(\frac{\gamma+1}{2}\right)}\right]^{\frac{\gamma+1}{2(\gamma-1)}}} \longrightarrow (6)$$

Equation of Area-Mach number relation.

(Arbitrary Area to choked Area in Duct).

* Graph of $\frac{A}{A^*}$ and M:

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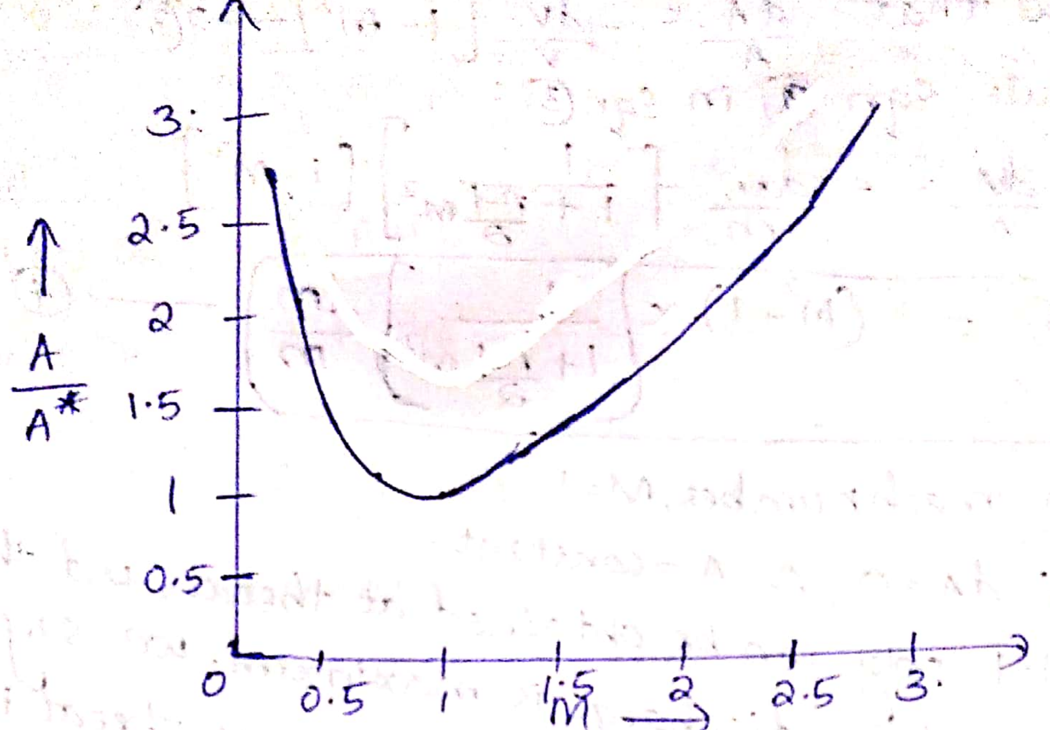
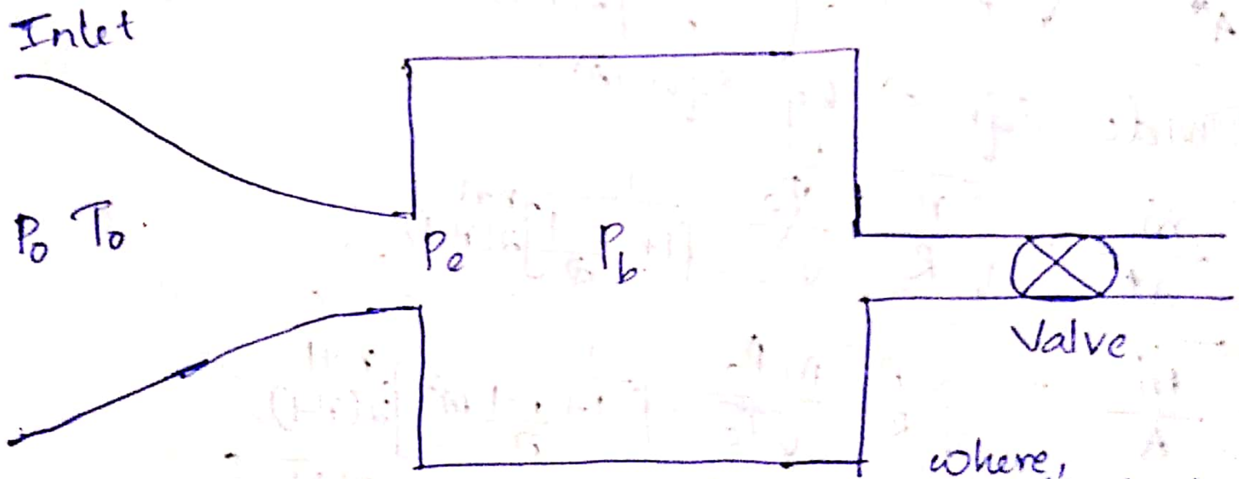


Double-valued curve

Other than $\frac{A}{A^*} = 1$, for any other values we get 2 values for a Mach number. (One is subsonic & other is supersonic)

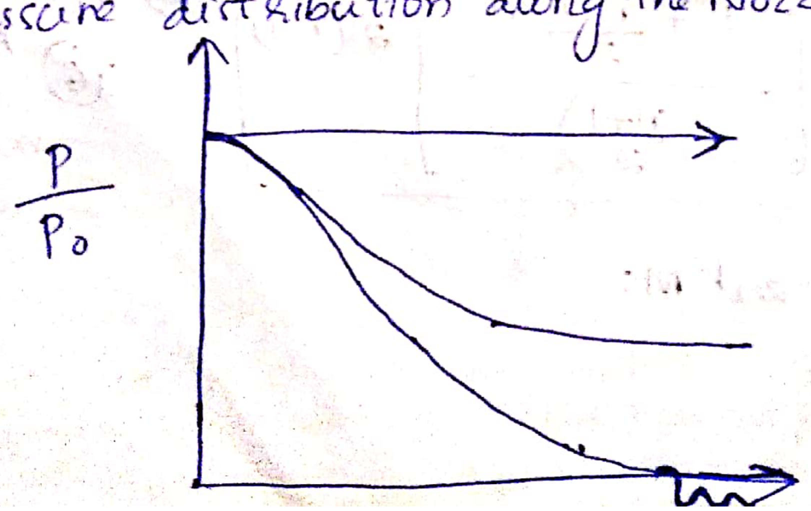
$\therefore$ Supersonic flow is diverging.

Pressure Distribution and choking in a Converging Nozzle:



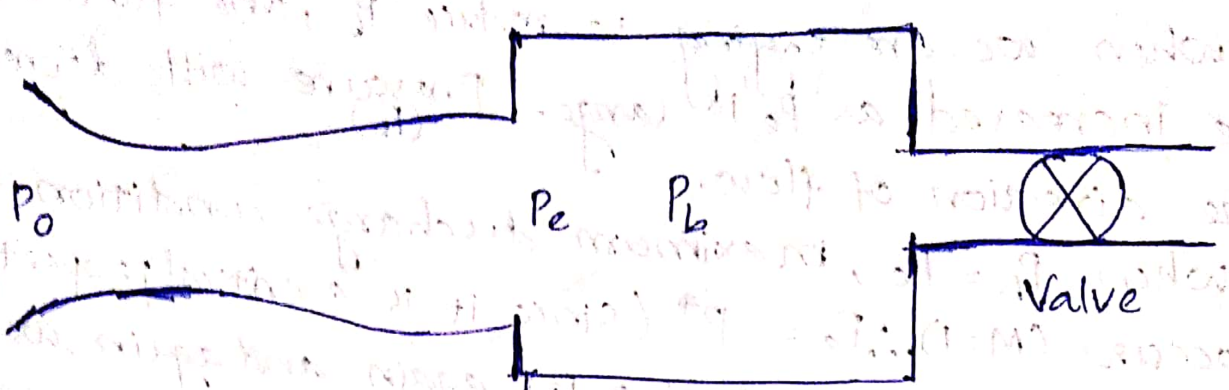
where,
P_b = back pressure
P_e = exit pressure
P_0 = stagnation pressure

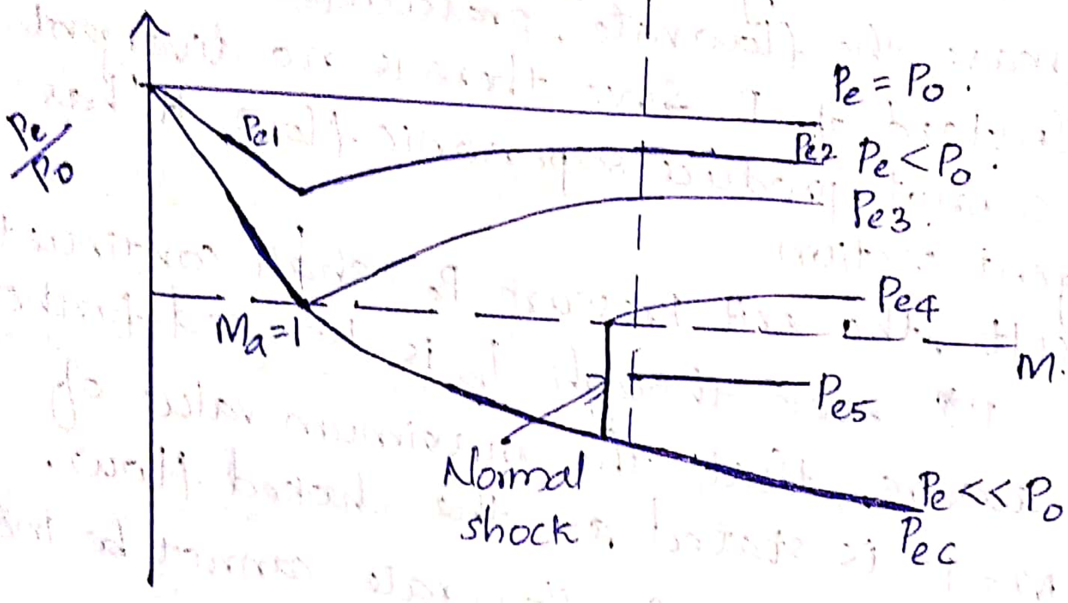
Pressure distribution along the Nozzle:



→ By adjusting the valve the value of P_b can be changed.
→ when $P_0 = P_e = P_b$, there will be no flow.
→ when we are trying to reduce 'P_b', the flow rate will be increased, as 'P_e' is large. Pressure (P_b) will decrease in the direction of flow.
→ when $P_b = P_e$, maximum discharge condition will occur. ($M=1$) $\therefore P_e = p^*$ (since it is a critical property).
→ when we are reducing P_b again and again, we does not increase the flowrate, pressure(P_e) remains constant. Since there is no divergent section, we cannot produce supersonic flow. It has only convergent section.
→ As a result, the exit Pressure P_e shall continue to remain at p^* even though P_b is lowered further.
→ we are aware that the maximum value of $\dot{m}/A$ at $M=1$ is stated as the choked flow.
→ with a given nozzle, the flow rate cannot be increased further.
→ However for P_b less than p^*, the flow leaving the nozzle has to expand to match the lower back pressure.
→ This expansion process is three-dimensional and the pressure distribution cannot be predicted by one-dimensional theory.
→ Experiments reveal that a series of shocks form in the exit stream, resulting in an increase in entropy.

(*) Converging-Diverging Nozzle: (Delaval Nozzle)





Nozzle is used to transfer pressure energy to kinetic energy. C.D Nozzle is used to produce Supersonic flows. If it does not have expanding portion, that particular nozzle cannot produce supersonic flow. At throat, M=1. i.e., sonic condition. Therefore throat is choked and thus it has critical properties.

From eq ⑥, we can say that $\frac{A}{A^*}$ is a function of mach number (M) only. (local area to sonic throat area)

(i) If $A < A^*$, It is physically not possible in an isentropic flow.

(ii) If $A > A^*$, then we have 2 values of mach number, one is subsonic and other is supersonic.

***Equations (Isentropic flow Mach Number Relationships):-**

(i) $\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$

Stagnation to static T

(ii) $\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$

Stagnation to static 'p'

(iii) $\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$

Stagnation to static 'ρ'.

$$\left\{ \frac{T}{T_1} = \left(\frac{\rho}{\rho_1}\right)^{\gamma - 1} = \left(\frac{P}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} \right\}$$

If we know a particular mach number, then we can calculate P, T, ρ using the above equations.

[In nozzle, P, T as well as ρ is decreasing continuously throughout as area is decreasing].

We designed a de-laval nozzle for a particular mach number and kept in still atmosphere, the air flow will not get accelerated. To accelerate the air, there should be some pressure gradient.

* $P_0 \neq P_e$

* $P_0 > P_e$ for generating a flow.

i.e., $\frac{P_e}{P_0} < 1$.

* If we need a supersonic velocity.

when $\frac{P_e}{P_0} \leq 0.528$

the contracting portion of the nozzle accelerate the flow upto mach number 1. The diverging portion further accelerates the flow beyond mach number 1.

At throat M=1 (It is choked).

→ At inlet section $\frac{P_{throat}}{P_0} < 0.528$ (Converging section)

→ At diverging section $\frac{P_e}{P_{throat}} < 1$, it can produce supersonic flow.

C-D Nozzle can experience a supersonic flow from the downstream of the throat to the exit. At exit there is a normal shock which is created by pressure difference.

(i) when $P_e = P_0$, there is no flow.

(ii) If value of P_e is slightly reduced than P_0, then we get a flow at low subsonic speed through the nozzle.

(iii) If P_e is again reduced, the pressure gradient will be stronger than previous one ∴ the flow acceleration will be large. Mach number will also be large.

(iv) Again we are reducing the exit pressure to P_{e3}. At this condition mach number will be equal to one (sonic condition). When this $P_{e3} < P^*$, it will produce supersonic flow at divergent section. If $P_{e3} > P^*$, the flow will get decelerated (ie, subsonic flow).

(v) When pressure is reduced lesser than P_{e3}, it is not at all effected in the convergent section. Mass flow remains constant in convergent section when $P_e < P_{e3}$. Their it is frozen.

(P_{e3} – pressure corresponding to choked throat)

(vi) But at divergent section there will be changes when $P_e < P_{e3}$. When we are reducing the pressure from P_{e3}

to P_{ec}, we will get a shockfree supersonic flow at divergent section at this particular pressure (P_{ec}). It is obtained due to isentropic expansion.

(vii) Now we are considering the pressuring range between P_{e3} and P_{ec}, there will be a normal shockwave existing in the inside of the divergent section. flow behind the normal shock is subsonic flow. If it is subsonic, pressure will get increased to P_{e4} at the exit.

(viii) Now we are reducing the pressure lesser than P_{e4} to P_{e5} (static pressure corresponding to designed mach number of Nozzle).
So the normal shock moves downstream.

(ix) Again we are reducing the pressure from P_{e5}
Inbetween P_{e5} and P_{ec}, the pressure is known as back pressure (P_b) (Pressure of the ambient atmosphere in which the flow is discharged).

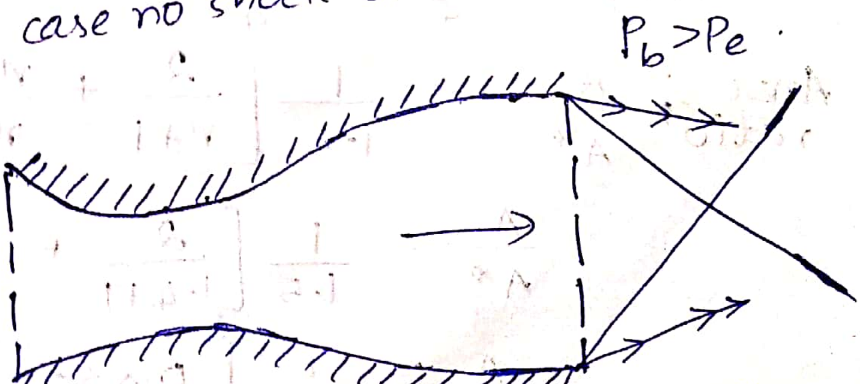
3 conditions are there at Nozzle exit:

(i) when $P_b > P_e$, oblique shock will be attached.

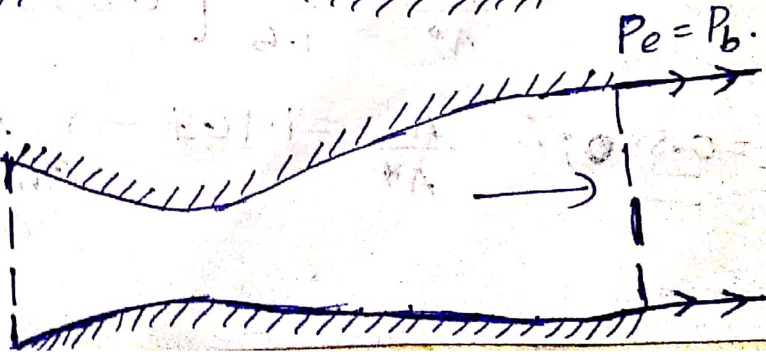
(ii) when $P_e > P_b$, there will be additional expansion occur. So the expansion waves are attached at exit.

(iii) $P_e = P_b$, then it is perfectly expanded at exit of the nozzle; in this case no shock-cell formation is expected.

(i) Over-expanded Nozzle

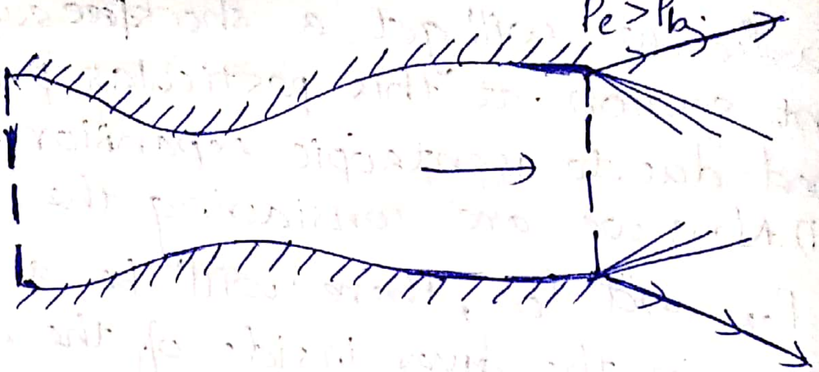


(iii) Perfectly expanded Nozzle



Scanned with CamScanner

(iii) Under-expanded Nozzle.



* Problems:

1) A De-laval Nozzle has to be designed for an exit mach number of 1.5 with an exit diameter of 200mm. find the required ratio of throat area to exit area, the reservoir conditions are given as: $P_0 = 1$ atm.
$T_0 = 20°C$.

find also the maximum mass flow rate through the nozzle. what will be the exit pressure and the temperature.

Ans: we know,

$$\text{Area ratio}, \frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{\gamma-1}{2}M^2}{\left(\frac{\gamma+1}{2}\right)}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad \text{and } \gamma = 1.4.$$

Given:

$M = 1.5$

$$\text{Exit Area}, A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314\,m^2$$

$$\text{Area ratio}, \frac{A}{A^*} = \frac{1}{1.5}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A}{A^*} = \frac{1}{1.5}\left[\frac{2}{1.4+1} + \frac{1.4-1}{1.4+1}M^2\right]^{\frac{1.4+1}{2(1.4-1)}}$$

$$\frac{A}{A^*} = \frac{1}{1.5}\left[0.833 + 0.375\right]^{3} = 1.176$$

$$\frac{A^*}{A} = 0.850; \quad \therefore \frac{A}{A^*} = 1.176 \implies A^* \text{(Throat area)} = \frac{0.0314}{1.176} = 0.0267\,m^2$$

Stagnation pressure, $P_0 = 1\,atm = 1.01325$ bar.

Stagnation temperature, $T_0 = 20°C = 293\,K$.

By Isentropic flow mach number relationship :

$$\frac{P_0}{P} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}} = \left[1 + (0.2 \times 1.5^2)\right]^{\frac{1.4}{1.4-1}}$$
$$= 3.6710$$

$$\frac{T_0}{T} = \left[1 + \frac{\gamma-1}{2} M^2\right] = 1 + (0.2 \times 1.5^2) = 1.45$$

$$\therefore \frac{P_0}{P} = 3.6710 \Rightarrow P = \frac{P_0}{3.6710} = \frac{1.01325}{3.6710} = 0.2760 \text{ bar}$$

$$\frac{T_0}{T} = 1.45 \Rightarrow T = \frac{T_0}{1.45} = \frac{293}{1.45} = 202.068\,K.$$

Maximum mass flow rate through the nozzle,

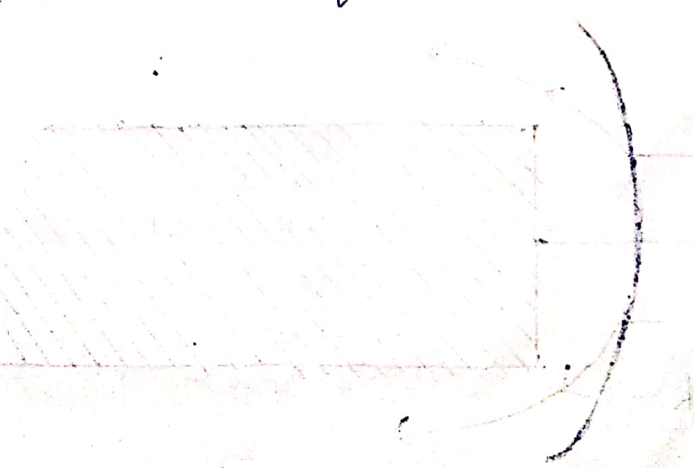
$$\dot{m} = A^* P_0 \sqrt{\frac{\gamma}{R T_0}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\dot{m} = 0.0268 \times 1.01325 \sqrt{\frac{1.4}{287.04 \times 293}\left(\frac{2}{1.4+1}\right)^{\frac{1.4+1}{2(0.4)}}}$$

$$\therefore \dot{m} = 0.00006411603\ \frac{kg}{s} \text{ or } 6.411 \times 10^{-5}\ kg/s$$

→ Topics

* Continuity, momentum and Energy equation for steady 1D-flow ~~and~~
* Compressible Bernoulli's equation.

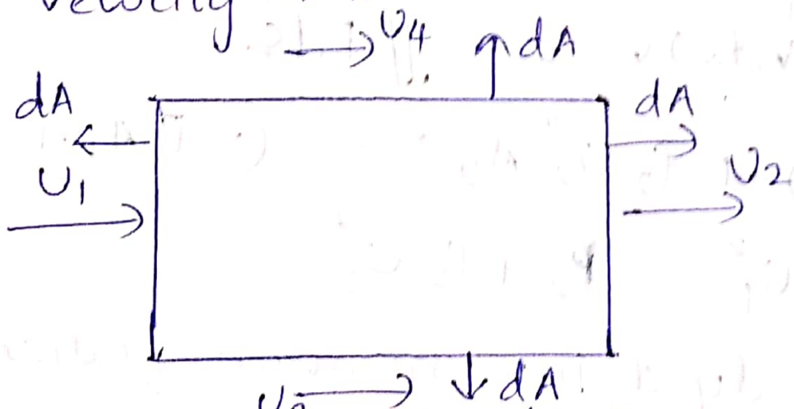


* MODULE - 1 (Continuation).

→ Assumptions for steady 1-D form of continuity, momentum and energy equations:

(i) The flow is steady.
(ii) The flow is one dimensional.
(iii) There are no body forces in the system.
(iv) The flow is inviscous.

* Consider a rectangular control volume,
$u \rightarrow$ velocity in x direction.



Consider continuity Equation;

$$-\oiint \rho v\, ds = \frac{\partial}{\partial t}\iiint \rho\, dv$$

Since the flow is steady $\frac{\partial}{\partial t}\iiint \rho\, dv = 0$.

$$\therefore\ -\oiint \rho v\, ds = 0$$

Since the flow is one-dimensional, we are considering only x component,

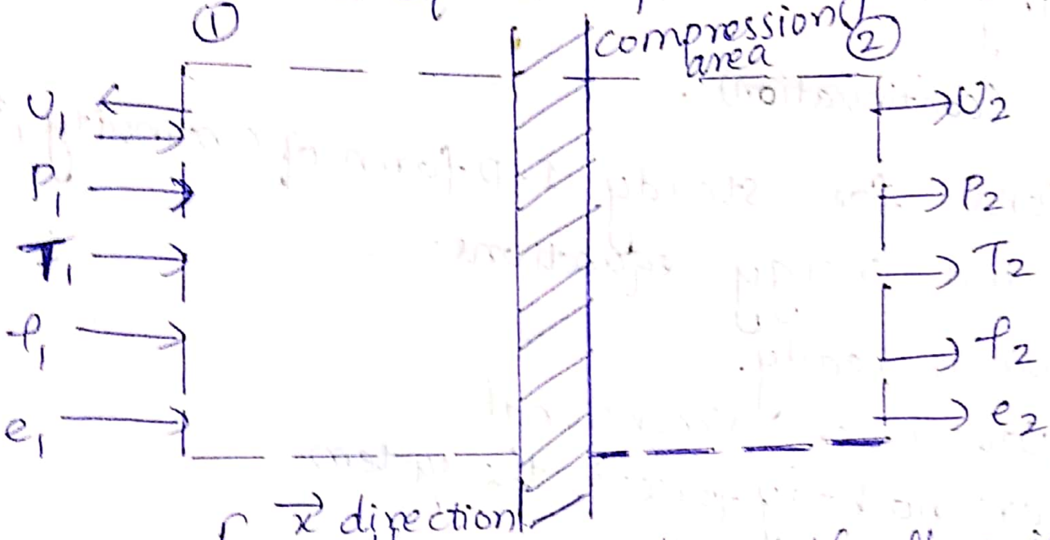
$$-\oiint \rho u \, dA = 0$$

$$\rho_1 U_1 A_1 + 0 - \rho_2 U_2 A_2 + 0 = 0.$$

$$\boxed{\rho_1 U_1 A_1 = \rho_2 U_2 A_2}$$

This is steady 1-D continuity equation.

* Momentum Equation for steady 1 D flow:



from the assumptions; the flow is steady and have no body forces and viscosity. So that the integral momentum equation will be reduced into:

$$\oiint (\rho V ds) V = -\oiint P ds.$$

$$-(\rho_1 U_1^2 A_1) + \rho_2 U_2^2 A_2 = -(-P_1 A_1 + P_2 A_2)$$

$$P_1 + \rho_1 U_1^2 = P_2 + \rho_2 U_2^2$$

This is steady 1-D momentum equation.

* Energy Equation for steady 1 D flow

$$\iiint_{\vartheta} \dot{q} \rho d\vartheta - \oiint_S PV \cdot ds + \iiint_{\vartheta} \rho (f \cdot V) d\vartheta$$

$$= \oiint_S (\rho V \cdot ds) \left(e + \frac{V^2}{2}\right) + \frac{\partial}{\partial t} \iiint_{\vartheta} \rho \left(e + \frac{V^2}{2}\right) d\vartheta.$$

By applying assumptions in the energy equation; such that the flow is steady, 1 D and of no body forces. Therefore the moment eqn can reduce into energy equation:

$$\iiint_V \dot{q}\, \rho\, dv.$$

Since the term represents total energy added to the system. For convenience it can be denoted as '$\dot{Q}$' where '$\dot{Q}$' is the total heat added to the system. Therefore, energy eqn will be equal to:

$$\dot{Q} - \iint P V \cdot ds = \iint \rho\left(e + \frac{V^2}{2}\right) V \cdot ds$$

$$\dot{Q} - (-P_1 U_1 A + P_2 U_2 A) = -\rho_1\left(e_1 + \frac{U_1^2}{2}\right) U_1 A + \rho_2\left(e_2 + \frac{U_2^2}{2}\right) U_2 A.$$

→ Dividing by A:

$$\frac{\dot{Q}}{A} + P_1 U_1 + \rho_1\left(e_1 + \frac{U_1^2}{2}\right) U_1 = P_2 U_2 + \rho_2\left(e_2 + \frac{U_2^2}{2}\right) U_2$$

Now divide the eqn with L.H.S of the eqn by $\rho_1 U_1$ and R.H.S of the eqn by $\rho_2 U_2$.

$$\frac{\dot{Q}}{A \rho_1 U_1} + \frac{P_1}{\rho_1} + e_1 + \frac{U_1^2}{2} = \frac{P_2}{\rho_2} + e_2 + \frac{U_2^2}{2}.$$

$$\boxed{\dot{q} + h_1 + \frac{U_1^2}{2} = h_2 + \frac{U_2^2}{2}}$$

* Bernoulli's equation for Compressible flow:

The Euler's equation is:

$$\frac{dP}{\rho} + v\,dv + g\,dz = 0.$$

By integrating eulers eqn we will get bernoulli's eqn.

$$\int \frac{dP}{\rho} + \int v\,dv + \int g\,dz = 0.$$

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = C.$$

→ Case I: Isothermal flow:

Temperature will be constant for isothermal flow:

$$\frac{p}{\rho} = C_1 \qquad \left(\text{From } P = \rho RT \Rightarrow \frac{P}{\rho} = C\right)$$

$$\rho = \frac{P}{C_1}$$

$$\int \frac{dP}{\rho} = C_1 \int \frac{dP}{P} = C_1 \ln P = \frac{P}{\rho} \ln P.$$

From the condition of isothermal $C_1 = \frac{P}{\rho}$.

$\therefore$ Bernoulli's eqn of compressible isothermal flow will be equal to:

$$\boxed{\frac{P}{\rho} \ln P + \frac{V^2}{2} + gz = C}$$

& by $\div$ by g, we get:

$$\frac{P}{\rho g} \ln P + \frac{V^2}{2g} + z = C.$$

Case II: Adiabatic flow.

$$\frac{P}{\rho^n} = C_2$$

where 'n' is adiabatic index.

$$\therefore \rho = \frac{P^{1/n}}{C_2^{1/n}}.$$

$$\therefore \int \frac{dP}{\rho} = \int C_2^{1/n} P^{-1/n} \, dP$$

$$= \frac{n}{n-1} \frac{P}{\rho}$$

$$\Rightarrow C_2^{1/n} P^{-1/n} = \frac{1}{\rho}$$

$$\Rightarrow C_2^{1/n} = \frac{P^{1/n}}{\rho}$$

$$\Rightarrow C_2^{1/n} \left[\frac{P^{1-1/n}}{1-1/n}\right]$$

$$= \frac{P^{1/n}}{\rho} \left[\frac{P^{1-1/n}}{1-1/n}\right]$$

$$= \frac{P}{\rho\left[1-\frac{1}{n}\right]}$$

$$= \frac{P}{\rho\left[\frac{n-1}{n}\right]}$$

$$= \frac{n}{n-1}\left[\frac{P}{\rho}\right]$$

$\therefore$ Bernoulli's eqn for compressible adiabatic flow will be

$$\boxed{\left(\frac{n}{n-1}\right)\frac{P}{\rho} + \frac{V^2}{2} + gz = C}$$

& by $\div$ by g, we get:

$$\left(\frac{n}{n-1}\right)\frac{P}{\rho g} + \frac{V^2}{2g} + z = C$$